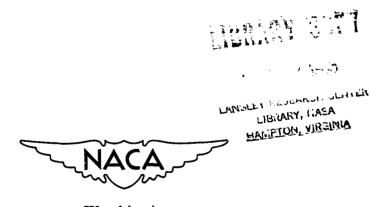
# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3200

STRESS ANALYSIS OF CIRCULAR SEMIMONOCOQUE CYLINDERS
WITH CUTOUTS BY A PERTURBATION LOAD TECHNIQUE

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#### SUMMARY

A method is presented for analyzing the stresses about a cutout in a circular cylinder of semimonocoque construction. The method involves the use of so-called perturbation solutions which are superposed on the stress distribution that would exist in the structure with no cutout in such a way as to give the effects of a cutout. The method can be used for any loading case for which the structure without the cutout can be analyzed and is sufficiently versatile to account for stringer and shear reinforcement about the cutout.

#### INTRODUCTION

Stresses near a cutout in a semimonocoque shell can be much higher than the stresses in the uniform shell some distance away from the cutout. The stress distribution in the neighborhood of cutouts in circular semimonocoque cylinders is significant in the design of fuselages near large openings such as doors and in the determination of the most efficient reinforcement to be used about these openings.

Some previous investigations relating to the problem of stress analysis of cylindrical semimonocoque shells with cutouts were reported in references 1 to 3. In reference 4, Cicala discussed the limitations of the analyses of references 1 to 3 - particularly the neglect of ring flexibility - and introduced the idea that the effect of a cutout can be reproduced by superposing certain perturbation stress states on the stresses which would occur in the shell without a cutout.

The problem discussed by Cicala in reference 4 is that of a cutout in an infinitely long circular cylinder of semimonocoque construction. Cicala's analysis is somewhat limited because it can be used only for loading conditions which produce stringer stresses longitudinally antisymmetric about the center line of the cutout (for example, torsion), and it cannot be used to determine the effects of coaming-stringer reinforcement. The present report is an extension of the approach of Cicala

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and presents a method of analysis which can be used with more generalloading conditions and with either shear or stringer reinforcement about the cutout.

The stress perturbation approach is applied to the stress analysis of sheet-stringer panels with cutouts in reference 5. Three basic unit-perturbation solutions were used as tools in this method of analysis. The analogous perturbation solutions for a circular semimonocoque cylinder have been developed in reference 6. The purpose of the present report is to explain the use of these perturbation solutions for the stress analysis of circular semimonocoque cylinders with cutouts.

#### SYMBOLS

A effective cross-sectional area of stringer

A' cross-sectional area of additional portion of reinforced stringer

$$B = \frac{E}{G} \frac{A}{bt} \frac{R^2}{L^2}$$

b arc distance between stringers

$$C = \frac{AR^6}{IL^3b}$$

E Young's modulus of elasticity

G shear modulus of elasticity

I moment of inertia of ring cross section

1,k indices for rings and bays

j, l indices for stringers and panel rows

L distance between rings

M<sub>1</sub>, M<sub>2</sub> applied moment and torque, respectively (see fig. 4)

m total number of stringers in cylinder,  $m \ge 3$ 

P magnitude of a concentrated perturbation load, lb

load in stringer j at ring station i  $\mathbf{p}_{\mathbf{i},\mathbf{i}}$ basic stringer load in stringer j at ring station i  $\overline{p}_{i,i}$  $p_{i,i}(k,l)$ load in stringer j at ring station i due to a unit concentrated perturbation load on stringer l at ring station k  $p_{1,1}[k,l]$ load in stringer j at ring station i due to a unit shear perturbation load about shear panel (k,1) magnitude of a shear perturbation load, lb/in. Q shear flow in shear panel (i,j) qij basic shear flow in shear panel (i,j) q<sub>i,i</sub>  $q_{i,i}(k,l)$ shear flow in shear panel (i,j) due to a unit concentrated perturbation load on stringer 1 at ring station k  $q_{ij}[k,l]$ shear flow in shear panel (i,j) due to a unit shear perturbation load about shear panel (k,1) radius to middle surface of sheet R magnitude of a distributed perturbation load, 1b S thickness of sheet t thickness of additional portion of a reinforced shear t١ panel, that is, doubler plate

# BASIC ASSUMPTIONS

A typical structure of the type to be discussed in this report is shown in figure 1. It consists of a thin-walled circular cylinder stiffened by stringers in the longitudinal direction and by rings in the circumferential direction. A cutout is located in a bay far from the ends of the cylinder (the theory is limited to cases where external restraints and free sections are a large distance from the cutout). The cutout which is 1 bay long may remove an arbitrary number of shear panels and portions of stringers.

The loading is considered to be such that buckling does not occur. Some possible loading conditions are shown in figure 1. Any other

loading condition is permissible if the stress distribution in the cylinder without the cutout is known.

The analysis is based on the following assumptions regarding the properties of the structure:

- (a) The stringers are uniform and equally spaced around the shell, and the sheet is of constant thickness.
- (b) The stringers carry only direct stress, and the sheet takes only shear stress which is constant within each bay; thus, stringer stresses vary linearly between adjacent rings.
- (c) The rings have a finite bending stiffness in their own planes, but they do not restrain longitudinal displacements of the stringers. The ring bending is inextensional.
- (d) The difference between the radius of the neutral axis of the ring and the radius of the middle surface of the sheet is negligible.

## PERTURBATION STRESS DISTRIBUTIONS

The tools for the method of analysis to be described are the stress distributions due to three types of loads, called perturbation loads, applied to an infinitely long circular cylinder with no cutout. One perturbation load consists of a concentrated force P imposed on one stringer of the shell at its intersection with a ring, the force acting in the direction of the stringer. This load is illustrated in figure 2(a) and is called the concentrated perturbation load. A second type, illustrated in figure 2(b), is called the distributed perturbation load and consists of a force S uniformly distributed along the portion of one stringer which extends between two adjacent rings, the force acting in the direction of the stringer. The third type, shown in figure 2(c), is called the shear perturbation load and consists of uniformly distributed forces per unit length Q applied along the stringers and rings that border one shear panel of the shell, the forces acting in such a way as to cause pure shear in that panel.

For each of the three perturbation loads, formulas were developed in reference 6 which give stringer loads in every stringer at each ring station and shear flows in each shear panel of the shell. By use of these formulas, tables of influence coefficients can be computed which give stringer loads and shear flows in the neighborhood of each perturbation load due to a unit magnitude of that load. Such tables for a cylinder having 36 stringers and various values of the sheet-stringer parameter B are presented as tables 1 to 3. The tables presented are

only for the limiting case of rigid rings (ring flexibility parameter C=0). In this report, tables 1 to 3 are used to make sample calculations illustrating the method of analysis for a shell with rigid rings. For a shell with flexible rings, the method of analysis is the same with the exception that tables which include the effect of ring flexibility will have to be used.

The tables are not limited in application to cylinders with 36 stringers. In general, the total stringer area can simply be redistributed into 36 fictitious stringers. The value of the parameter B is not changed by such a redistribution of stringer area. Then the tables can be thought of as presenting (a) the load which is taken by all of the normal-stress-carrying material up to 5° on either side of the location of a fictitious stringer and (b) the shear flows at points in the sheet halfway between fictitious stringers.

Table 1 contains the values of  $p_{i,j}$  and  $q_{i,j}L$  due to a concentrated perturbation load P=1 on stringer j=0 at ring station i=0. Table 2 contains the values of  $p_{i,j}$  and  $q_{i,j}L$  due to a distributed perturbation load of total magnitude S=1 on stringer j=0 between rings i=0 and i=1. Table 3 contains the values of  $p_{i,j}/L$  and  $q_{i,j}$  due to a shear perturbation load per unit length of magnitude Q=1 about shear panel (0,0). The positive senses of the perturbation loads are the senses shown in figure 2; stringer loads are assumed positive in tension, and shear flow is positive when an element of the sheat is loaded by shears which act in the positive sense of the shear perturbation load. The solutions for arbitrary locations of the perturbation loads are readily obtained from the tables by means of changes of indices.

The application of these perturbation loads and the stress distributions caused by them in the stress analysis of circular semimonocoque cylinders with cutouts is discussed in the following section. The perturbation solutions are exact only for infinitely long cylinders. However, in the solution of a cutout problem, the perturbation loads are applied in self-equilibrating groups in order not to disturb the overall equilibrium of the structure; therefore, the stresses due to the perturbation loads decay rapidly in the longitudinal direction. Consequently, the application of perturbation stress distributions for an infinitely long cylinder to a cylinder of finite length is justified if the vicinity of application of the perturbation loads is far from the ends of the cylinder.

#### METHOD OF ANALYSIS

#### Structure With No Reinforcement About Cutout

Application of perturbation loads.— Consider, first, a structure like that shown in figure 1 which has no reinforcement about the cutout. The stress distribution in such a shell can be thought of as a superposition of the stresses which would exist in the structure without a cutout and perturbation stress distributions which arise because of the cutout. The structure without a cutout is called herein the basic structure. The stress distribution which would exist in this structure is called herein the basic stress distribution. In the present report the basic stress distribution is assumed to be known. Then the problem of analyzing a structure with a cutout consists of the determination of the perturbation stress distributions to be superposed on the basic stresses in such a manner as to annihilate the effects of that portion of the basic structure which lies within the boundaries of the cutout. Finding the proper magnitudes of these perturbation stresses involves the solution of a system of simultaneous algebraic equations.

At the cutout boundary in the structure with the cutout, two conditions must be satisfied: (a) zero stringer load wherever a stringer is interrupted by the cutout and (b) no external shear forces acting on the portions of stringers and rings which border the cutout. By superposing concentrated and shear perturbation loads on the basic structure, the resultant stresses can be made to satisfy these conditions.

The method of analysis is as follows:

- (1) Find the stress distribution for the basic structure, that is, the cylinder without a cutout.
- (2) Place perturbation loads on the basic structure in the following manner: At each point where a stringer would be interrupted by the cutout, place a concentrated perturbation load; and, about each shear panel which would be removed by the cutout, place a shear perturbation load. For the case of a cutout removing three shear panels and interrupting two stringers, these perturbation loads are shown in figure 3.
- (3) With the use of the tables of influence coefficients, write a set of simultaneous algebraic equations which state the following conditions:
  - (a) At the points where-a stringer is to be interrupted by the cutout boundary, the resultant stringer load must vanish on the side of the boundary away from the cutout. This resultant stringer load is composed of the basic stringer load plus the stringer load due to all the perturbation loads.

(b) In each shear panel which is to be removed by the cutout, the basic shear flow plus the shear flow due to all the perturbation loads must be equal to the shear perturbation load applied to the portions of stringers and rings which border that given panel. Thus, the shear flow exerted by the shear panel on the portions of stringers and rings bordering it will exactly cancel the shear perturbation load applied to those same portions of stringers and rings.

(4) Solve the system of equations from step (3) for the magnitudes of the perturbation loads, and superpose the stress distributions due to these loads on the basic distribution. This procedure yields the stress distribution in the structure with cutout.

Upon completion of these four steps, the magnitudes of the perturbation loads on the basic structure have been adjusted so that simultaneous removal of that portion of the basic structure which lies within the cutout boundary and the perturbation loads themselves would not disturb the remainder of the structure. The perturbation loads are in equilibrium with the portion of the basic structure lying within the cutout boundary. The stresses outside the cutout boundary in the basic structure subjected to the actual external loading together with the perturbation loads are precisely the same as the stresses in the structure with the cutout subjected to the external loading alone.

Conditions 3(a) and 3(b) can be expressed mathematically by the following equations, respectively:

$$\sum_{\mathbf{k}} \sum_{l} P_{\mathbf{k}l} p_{\mathbf{i}\mathbf{j}}(\mathbf{k}, l) + \sum_{\mathbf{k}} \sum_{l} Q_{\mathbf{k}l} p_{\mathbf{i}\mathbf{j}}[\mathbf{k}, \underline{l}] + \overline{p}_{\mathbf{i}\mathbf{j}} = 0$$
 (1)

$$\sum_{\mathbf{k}} \sum_{l} P_{\mathbf{k}l} \mathbf{q_{ij}}(\mathbf{k}, l) + \sum_{\mathbf{k}} \sum_{l} Q_{\mathbf{k}l} \mathbf{q_{ij}} [\overline{\mathbf{k}}, \overline{l}] + \overline{\mathbf{q}_{ij}} = Q_{\mathbf{ij}}$$
 (2)

The unknowns are  $P_{kl}$ , the magnitude of the concentrated perturbation load on stringer l at station k, and  $Q_{kl}$ , the magnitude of the shear perturbation load about shear panel (k,l). The coefficients  $p_{i,l}(k,l)$ 

and  $q_{i,j}(k,l)$  are found in table 1 and the coefficients  $p_{i,j}[k,\overline{l}]$ 

and  $q_{i,j}[k,\overline{l}]$  are found in table 3. The summations in each case are extended over the appropriate perturbation loads. Equation (1) is written for each i,j where a stringer is to be interrupted by the

cutout and refers in each case to the stringer load as the point i,j is approached from within that portion of the structure lying outside the cutout boundary. Equation (2) is written for each i,j where a shear panel is to be removed by the cutout. The form of equations (1) and (2) is the same regardless of whether the rings are considered rigid or flexible.

This method of analysis may be applied to a cylinder having a cutout more than 1 bay long, but, in such a situation, the effects of
removing ring segments from the region within the cutout boundary are
neglected. In the rigid-ring case, such effects do not-exist if the
cut rings remain effectively rigid; in the flexible-ring case, the
effects of cutting a ring could, in principle, be taken into account
through the introduction of additional types of perturbation loads. It
is possible that even with flexible rings the effects of cutting a ring
are negligible in certain cases, but this would have to be verified by
further investigation.

Sample calculation. In order to illustrate the method of calculation, the cylinder shown in figure 4 is analyzed. A cutout which removes three shear panels and interrupts two stringers is located in the central bay. The required properties of the cylinder are taken as follows:

$$m = 36$$

$$A = 0.260 \text{ sq in.}$$

$$R = 15 in.$$

$$L = 12 in.$$

$$t = 0.051 in.$$

$$b = R \frac{2\pi}{36} = 2.62 in.$$

$$B = \frac{E}{G} \frac{A}{bt} \frac{R^2}{T^2} = 8.05$$

Suppose that the cylinder is loaded with the bending moment  $M_1$  and torque  $M_2$  shown in figure 4.

The perturbation load system for this problem is shown in figure 3. The concentrated perturbation loads are doubly symmetric about the cutout.

The shear perturbation loads are symmetric about panel row j=0. Let P represent the magnitude of each of the concentrated perturbation loads. Let  $Q_0$  represent the magnitude of the shear perturbation load about shear panel (0,0); and let  $Q_1$  represent the magnitude of the shear perturbation loads about shear panels (0,1) and (0,-1).

Equations (1) and (2) are written for this example by use of the tables of influence coefficients for B=8. Equation (1) for the stringer load condition in stringer j=1 at ring station i=1 is written with the aid of tables l(a) and 3(a)

$$-0.5000P + 0.0476P + 0.0895P + 0.1192Q_1L - 0.1192Q_0L - 0.0374Q_1L + \overline{p}_{11} = 0$$

where  $\bar{p}_{11}$  is the basic stringer load in stringer j=1 at station i=1. Because of symmetry the same equation results when equation (1) is written for stringer j=1 at station i=0 or for stringer j=0 at ring stations i=0 or i=1. Equation (2) for shear panel (0,0) is

$$-0.2262 \frac{P}{L} + 0.2262 \frac{P}{L} - 0.2262 \frac{P}{L} + 0.2262 \frac{P}{L} + 0.6986Q_{O} - 2(0.0629)Q_{1} + \overline{q}_{OO} = Q_{O}$$

where  $\overline{q}_{00}$  is the basic shear flow in shear panel (0,0). For shear panels (0,1) and (0,-1), equation (2) gives

$$-0.2262 \frac{P}{L} + 0.2262 \frac{P}{L} - 0.1368 \frac{P}{L} + 0.1368 \frac{P}{L} + 0.6986Q_{1} - 0.0629Q_{0} + 0.0119Q_{1} + \overline{Q}_{01} = Q_{1}$$

where  $\overline{q}_{Ol}$  is the basic shear flow in shear panel (0,1). These three equations in the three unknowns P,  $Q_O$ , and  $Q_I$  become

$$0.3629P + 0.1192Q_{0}L - 0.0818Q_{1}L = \overline{p}_{11}$$

$$0.3014Q_{0}L + 0.1258Q_{1}L = \overline{q}_{00}L$$

$$0.0629Q_{0}L + 0.2895Q_{1}L = \overline{q}_{01}L$$
(3)

For simplicity, let  $M_1 = M_2 = 100,000$  lb-in. In the present example, the basic stress distribution can be found from engineering beam theory which gives  $\overline{p}_{11} = 370$  pounds and  $\overline{q}_{00} = \overline{q}_{01} = 70.8$  lb/in. When these constants are introduced into the system of equations (3), the solution is

$$P = 1,020 lb$$

$$Q_0L = 1,750 \text{ lb}$$

$$Q_1L = 2,560 lb$$

Stringer loads and shear flows in the neighborhood of the cutout are obtained by superposing the effects of these perturbation loads on the basic stress distribution. For example, with the use of tables l(a) and 3(a) the stringer load at the intersection of ring i=0 and stringer j=2 is given by

$$P(0.0895 + 0.0511) + Q_1L(0.1192 + 0.0125) + Q_0L(0.0374) + \overline{P}_{02} = 545 + \overline{P}_{02}$$

The basic stringer load  $\overline{p}_{O2}$  equals 358 pounds. Therefore, the load in stringer j = 2 at ring i = 0 is 903 pounds. Other stringer loads at ring i = 0 are shown in figure 5(a). The shear flow in shear panel (-1,1) is given by

$$\frac{1}{L} \left[ P(0.2262 + 0.1368 + 0.0044 - 0.0360) + Q_{1}L(0.1357 - 0.0159) + Q_{0}L(0.0097) \right] + \overline{q}_{-1,1} = 55.1 + \overline{q}_{-1,1}$$

The basic shear flow  $\overline{q}_{-1,1}$  equals 70.8 lb/in. Thus, the shear flow in panel (-1,1) is 125.9 lb/in. Other shear flows in bay i=-1 are shown in figure 5(b), and in figure 5(c) are presented shear flows in the net section (bay i=0).

# Structure With Reinforcement About Cutout

Shear reinforcement. The method of analysis is easily extended to problems where shear panels are reinforced in the neighborhood of the cutout. Suppose that some of the shear panels around the cutout are reinforced by the addition of a certain thickness of sheet (i.e., a doubler plate). Then, the procedure consists of adding shear perturbation loads to each of these shear panels in the basic structure. On the doubler plates is placed the same shear perturbation load except with opposite sign. Then, for each reinforced shear panel, an equation is written which states the requirement that the shear stress in the shear panel of the basic structure shall equal the shear stress in the doubler plate used to reinforce that panel. When this condition is satisfied, the loaded doubler plates can conceptually be inserted into the basic structure without disturbing continuity. The shear perturbation loads on the doubler plates cancel the shear perturbation loads on the basic structure.

As an example, consider for simplicity the cylinder shown in figure 4 loaded only with bending moment  $\rm M_1$ . The most highly loaded shear panels are those indicated by the vertical hatching in figure 6. Suppose, now, that these shear panels are reinforced by the addition of plates of thickness t' to the skin of thickness t so that the total thickness in these shear panels is t + t'. The perturbation load system to be placed on the basic structure is shown in figure 7. The four doubler plates of thickness t' are shown as free bodies in figure 7. The shear perturbation loads applied to them are of the same magnitude as those applied to the basic portions of the reinforced shear panels, but are opposite in sign. The conditions that must be satisfied are:

- (a) The stringer load is zero in stringers j = 0 and j = 1 at stations i = 0 and i = 1 as each of these points is approached from the structure outside of the cutout.
- (b) The shear flow in shear panels (0,-1), (0,0), and (0,1) cancels any shear perturbation load applied about these panels. (In this example, no shear is developed in the shear panels of bay i=0 and this condition is automatically satisfied.)
- (c) The shear stress in each of the shear panels (1,1), (1,-1), (-1,1), and (-1,-1) in the basic structure must equal the shear stress in the corresponding doubler plate.

Condition (a), which must hold where stringers j=0 and j=1 are interrupted by the cutout, is expressed by a single equation because of symmetry:

$$(-0.5000 + 0.0476 + 0.0895)P + (-0.1192 - 0.0374 + 0.0067 - 0.0118)QL + \overline{P}_{11} = 0$$

where P and Q are the magnitudes of the concentrated and shear perturbation loads, respectively, and  $\overline{p}_{11}$  is the basic stringer load. The condition in shear panel (1,1) that the shear stress in the basic portion of the sheat equals the shear stress in the doubles related (1,2).

The condition in shear panel (1,1) that the shear stress in the basic portion of the sheet equals the shear stress in the doubler plate (condition (c)) is expressed as

where t is the thickness of the basic portion of the shear panel and t' is the thickness of the doubler plate. Because of symmetry, the same equation expresses condition (c) for the other three reinforced shear panels. These equations become

$$0.3629P + 0.1617QL = \overline{p}_{11}$$

$$-0.3314P + \left(\frac{t}{t^1} + 0.6851\right)QL = 0$$

For a given value of t/t' and for a given magnitude of  $M_1$  (so that  $\overline{p}_{11}$  can be computed), this system of equations can be solved for P and Q, and the stress distributions due to these perturbation loads can then be superposed on the basic stress distribution to give the stresses about the cutout.

Stringer reinforcement.— The method of analysis is also easily extended to problems where stringers are reinforced in the neighborhood of the cutout. For example, suppose the coaming stringers in the structure shown in figure 4 have reinforcement of constant cross-sectional area extending 1 bay on either side of the cutout. This coaming-stringer reinforcement is illustrated in figure 8. Let the area of the added reinforcing portion of a coaming stringer be A' so that the total area of the reinforced portion of the stringer is A + A'. It is assumed that the stringer load is abruptly transmitted into the added portion of the reinforced coaming stringer so that the stress is always given by the force divided by the cross-sectional area.

The perturbation load system to be placed on the basic structure is shown in figure 9. The added reinforcing portions of the coaming stringers are shown as free bodies in figure 9 with the proper perturbation loads applied to them. The conditions that must be satisfied are:

(a) The stringer load is zero in stringers j=0 and j=1 at stations i=0 and i=1 as each of these points is approached from the structure outside of the cutout.

- (b) The shear flow in shear panels (0,-1), (0,0), and (0,1) cancels any shear perturbation load applied about these shear panels. (This condition is automatically satisfied in this example.)
- (c) The stress in the basic portions of the coaming stringers j=-1 and j=2 equals the stress in the added reinforcing portions at stations i=0 and i=1.
- (d) In the basic portions of the coaming stringers j = -1 and j = 2 at stations i = -1 and i = 2, when these points are approached from the side which is reinforced, the stress equals the stress at the ends of the added reinforcing portions of the coaming stringers.

Because of the symmetry in this structure, only three equations are required. The unknowns are  $P_1$  and  $P_2$ , the magnitudes of the concentrated perturbation loads, and S, the magnitude of the distributed perturbation loads. Condition (a), which must hold where stringer j=1 is interrupted by the cutout, is expressed with the use of tables 1(a) and 2(a) as follows:

$$(-0.5000 + 0.0476 + 0.0895)P_1 + (-0.0895 - 0.0511 - 0.0490 - 0.0475)P_2 + (-0.0727 - 0.0340 - 0.0629 - 0.0499)S +  $\overline{P}_{11} = 0$$$

The condition that the stringer stress in the basic portion of stringer j = 2 equals the stress in the added reinforcing portion at station i = 1 (condition (c)) is expressed as

Finally, the condition that the stress in the basic portion of stringer j=2, as the station i=2 is approached from the reinforced side, equals the stress at the ends of the added reinforcing member (condition (d)) is expressed as follows:

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These three equations become

$$0.3629P_{1} + 0.2371P_{2} + 0.2195S = \overline{p}_{11}$$

$$-0.1406P_{1} + \left(\frac{A}{A'} + 0.1773\right)P_{2} + \left(\frac{A}{A'} + 0.3065\right)S = \overline{p}_{12}$$

$$0.0441P_{1} + \left(\frac{A}{A'} + 0.5853\right)P_{2} - 0.1222S = \overline{p}_{22}$$

When A/A' is known and the magnitude of the external moment  $\rm M_1$  is known so that the basic stringer loads  $\rm \bar{p}_{11}$ ,  $\rm \bar{p}_{12}$ , and  $\rm \bar{p}_{22}$  can be computed, this system of equations can be solved for the unknowns  $\rm P_1$ ,  $\rm P_2$ , and S. Superposition of the stresses due to these perturbation loads on the basic stress distribution yields the stresses about the cutout.

In this example the basic stringer loads do not vary in the longitudinal direction, and the concentrated and distributed perturbation loads can be applied in pairs, equal in magnitude and opposite in sign, as shown in figure 9. However, in cases where the basic stringer loads do vary longitudinally, for example, when the shell is loaded in shear and bending, the concentrated and distributed perturbation loads in general will not be equal in magnitude. Furthermore, additional distributed perturbation loads will be necessary on the coaming stringers in bay i = 0. Now the stress conditions which were used in the example alone no longer provide a sufficient number of equations to determine the magnitudes of the perturbation loads. The required supplementary equations are found from the conditions of equilibrium obtained when the added reinforcing portions of the coaming stringers are considered as free bodies.

Comparison of results for reinforced and unreinforced structures. - Some calculated results for the problems of cutouts with reinforcement just discussed are compared with the results for the structure without reinforcement in the following table:

	Strin	nger load, lb, for	-
Intersection of ring and stringer	Structure without reinforcement	Structure with reinforced coaming stringers, A' = A	Structure with shear reinforcement, t' = t
(1,2) (1,3) (1,4) (1,5) (1,6)	501. 422 359 303 244	758 331 296 258 209	507 422 359 302 242

	She	ar flow, lb/in., for	-
Shear panel	Structure without reinforcement	Structure with reinforced coaming stringers, A' = A	Structure with shear reinforcement, t' = t
(1,0) (1,1) (1,2) (1,3) (1,4)	0 -28.1 -12.3 -5.6 -2.5	0 -27.3 3 .4 .5	0 -30.6 -13.3 -5.8 -2.3

The reinforced shear panels were assumed to have sheet twice as thick as the uniform sheet; the reinforced portions of the coaming stringers were taken to have twice the area of the uniform stringers. The applied bending moment  $M_1$  was taken as 100,000 lb-in.

The following comparison is noted for these illustrative examples: In the case of coaming-stringer reinforcement, the maximum stringer load is increased, but the maximum stringer stress is decreased (because stringer area is doubled), and the maximum shear flow is not appreciably changed. In the case of shear reinforcement, the maximum shear flow is increased only slightly so that maximum shear stress is considerably reduced, and stringer loads are not appreciably affected.

### CONCLUDING REMARKS

The method presented in this report facilitates stress analysis of circular semimonocoque cylinders with cutouts. It is most accurate in problems where the cutout is located far from external restraints and

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free sections. The loading may be any combination of torsion, bending, shear, axial loading, and, in fact, any loading for which the basic stress distribution is known. Reinforcement about the cutout can be taken into consideration.

The method of analysis is based on the superposition of certain perturbation stress distributions to give the effects of the cutout on the stress distribution which would exist in the cylinder without a cutout. The equations for the three necessary perturbation stress distributions are presented in NACA Technical Note 3199. Perturbation stress distributions in the form of tables of influence coefficients are presented in this report for a 36-stringer shell having rigid rings. The tables can be used for cylinders having any number of stringers by "relumping" the actual stringers into 36 fictitious stringers. These tables are used in the report to make sample-calculations which illustrate the analytical procedure. The procedure does not change in the case for which the cylinder has flexible rings except that tables of influence coefficients which include the effects of ring flexibility would have to be used.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., June 16, 1954.

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Table 1.- Load distribution due to a unit concentrated perturbation load  $\text{on stringer} \quad j \ = \ 0 \quad \text{at station} \quad i \ = \ 0$ 

$$[c = 0; m = 36]$$

(a) 
$$B = 8$$

		St	ringer loa	d, p <sub>ij</sub> , a	t station	_	
j	i = 0	i = 1	i = 2	1 = 3	1 = 4	1 = 5	i = 6
0 1 2 3 4 5 6 7 8 9 0 1 1 2 3 4 5 6 1 7 8 9 1 1 2 3 4 1 5 6 1 7 1 8	0.5000	0.0476 .0895 .0511 .0330 .0232 .0172 .0130 .0097 .0047 .0026 .0007 0010 0024 0053 0057 0059	0.0565 .0490 .0475 .0402 .0329 .0266 .0212 .0165 .0123 .0010 .0056 .0072 .0056 .0072 .0094	0.0459 .0457 .0429 .0394 .0349 .0300 .0250 .0202 .0154 .0110 .0067 .0028 008 008 008 008 008 00111 0115	0.0437 .0430 .0414 .0387 .0352 .0351 .0266 .0219 .0171 .0124 .0078 .0035 0040 0070 0094 0112 0126	0.0426 .0421 .0406 .0383 .0352 .0315 .0273 .0227 .0180 .0131 .0084 .0039 0040 0072 0098 0117 0128 0132	0.0421 .0416 .0403 .0381 .0352 .0316 .0276 .0231 .0184 .0135 .0087 .0041 0002 0040 0073 0100 0132 0136

		Shear i	?low, q <sub>ij</sub>	L, at stat	ion -	
j	i = 0	i = 1	i = 2	i = 3	i = 4	i = 5
0 1 2 3 4 5 6 7 8 9 0 1 1 2 3 4 1 5 6 1 7	0.2262 .1368 .0856 .0527 .0294 .0122 0105 0175 0222 0248 0255 0246 0222 0186 0140 0087 0029	-0.0044 .0366 .0396 .0324 .0227 .0133 .0052 0016 0106 0129 0140 0140 0129 0110 0083 0052 0018	0.0053 .0087 .0133 .0141 .0086 .0047 .0010 0064 0073 0076 0072 0062 0048 0030 0010	0.0011 .0038 .0053 .0059 .0056 .0045 .0029 .0012 0030 0037 0040 0039 0034 0026 0017	0.0006 .0015 .0023 .0026 .0026 .0022 .0016 .0008 .0000 0018 0014 0020 0020 0018 0014 0019 0003	0.0002 .0007 .0010 .0012 .0012 .0011 .0008 .0004 .0000 0003 0006 0009 0010 0010 0009 0007 0005 0002

TABLE 1.- LOAD DISTRIBUTION DUE TO A UNIT CONCENTRATED PERTURBATION LOAD

ON STRINGER j = 0 AT STATION i = 0 - Continued

$$[c = 0; m = 36]$$

j i=0 i=1 i=2 i=3 i=4	1 = 5	i = 6
	O OLOT	
0         0.5000         0.1518         0.0852         0.0636         0.0518           1         0         .0866         .0711         .0588         .0518           2         0         .0374         .0488         .0484         .0462           3         0         .0209         .0331         .0380         .0393           4         0         .0137         .0237         .0296         .0326           5         0         .0098         .0177         .0232         .0267           6         0         .0072         .0134         .0181         .0214           7         0         .0054         .0100         .0138         .0167           8         0         .0038         .0073         .0102         .0125           9         0         .0025         .0049         .0069         .0086           10         0         .0013         .0027         .0040         .0051           11         0         .0003         .0008         .0013         .0019           12         0        0006        0009        0010        0031           14         0        0021        0037<	0478 0443 0394 0394 0288 0236 0188 0143 0100 0061 0024 - 0009 - 0037 - 0062 - 0081 - 0095 - 0104 - 0107	0.0463 .0454 .0429 .0392 .0348 .0300 .0251 .0203 .0156 .0111 .0068 .0029 0007 0039 0066 0087 0103

		Shear	flow, qij	L, at stat	ion -	
j	i = 0	i = 1	1 = 2	i = 3	1=4-	1=5
0 1 2 3 4 5 6 7 8 9 0 1 1 2 3 4 1 5 6 1 7	0.1741 .0875 .0501 .0292 .0155 .0057 0016 0168 0133 0146 0149 0143 0128 0128 0107 0080 0050	0.0333 .0489 .0375 .0253 .0153 .0074 .0012 0034 0069 0106 0108 0098 0098 0062 0039	0.0108 .0231 .0234 .0186 .0127 .0072 .0025 0013 0042 0063 0080 0074 0080 0048 0030	0.0048 .0118 .0140 .0126 .0096 .0061 .0028000100250042005900590059004700360028	0.0025 .0065 .0084 .0083 .0068 .0047 .0025 .0004 0014 0027 0042 0043 0041 0035 0027 0017	0.0014 .0038 .0052 .0054 .0047 .0035 .0021 .0006 0025 0030 0030 0030 0026 0020

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TABLE 1.- LOAD DISTRIBUTION DUE TO A UNIT CONCENTRATED PERTURBATION LOAD  $\hbox{ON STRINGER} \quad {\tt j} \ = \ \hbox{O} \quad \hbox{AT STATION} \quad {\tt i} \ = \ \hbox{O} \ - \ \hbox{Continued}$ 

$$[c = 0; m = 36]$$

(c) B = 100

j		St	ringer loa	d, p <sub>ij</sub> , a	t station	<b>-</b>	
J	i = 0	1 = 1	1 = 2	i = 3	i = 4	i = 5	i = 6
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	0.5000 0 0 0 0 0 0 0 0 0 0	0.2548 .0699 .0241 .0124 .0079 .0055 .0041 .0030 .0014 .0007 .0002 0004 0008 0015 0019 0020	0.1528 .0800 .0391 .0225 .0149 .0106 .0079 .0058 .0042 .0028 .0015 .0004 0006 0015 0023 0029 0037	0.1062 .0750 .0460 .0297 .0206 .0151 .0113 .0084 .0061 .0040 .0022 .0006 0008 0021 0032 0040 0046 0050	0.0825 .0678 .0483 .0250 .0189 .0144 .0108 .0079 .0053 .0030 .0009 0009 0025 0039 0058 0064	0.0692 .0615 .0484 .0369 .0219 .0170 .0129 .0055 .0064 .0037 .0012 0010 0029 0045 0058 0068 0074	0.0611 .0566 .0475 .0383 .0306 .0243 .0191 .0147 .0109 .0075 .0044 .0015 0032 0050 0065 0083 0085

1		Shear	flow, q <sub>ij</sub> L	, at stati	on -	
J	i = 0	1 = 1	i = 2	1 = 3	i = 4	1 = 5
0 1 2 3 4 5 6 7 8 9 10 11 2 13 14 15 16 17	0.1226 .0527 .0286 .0162 .0083 .0028 0013 0043 0085 0085 0087 0087 0087 0047 0029 0010	0.0510 .0409 .0259 .0157 .0087 .0036 .0051 .0064 .0074 .0064 .0064 .0054 .0051	0.0233 .0283 .0214 .0142 .0085 .0040 .0066 .0053 .0060 .0063 .0061 .0055 .0047 .0055 .0047	0.0119 .0191 .0168 .0123 .0079 .0041 .0013 0031 0043 0050 0053 0052 0048 0040 0030 0019	0.066 0.129 0.129 0.129 0.070 0.070 0.0014 0.0035 0	0.0041 .0089 .0098 .0083 .0060 .0015 0003 0018 0028 0035 0038 0036 0030 0022 0014 0005

TABLE 1.- LOAD DISTRIBUTION DUE TO A UNIT CONCENTRATED PERTURBATION LOAD ON STRINGER j=0 AT STATION i=0 - Continued  $\begin{bmatrix} C=0; & m=36 \end{bmatrix}$ 

(d) B = 300 . . . . . .

		St	ringer loa	d, p <sub>ij</sub> , a	t station	_	
Ĵ	i = 0	1 = 1	i = 2	1 = 3	1 = 4	i = 5	i = 6
0	0.5000	0.3354	0.2366	0.1756	0.1370	0.1116	0.0945
1 2	0	.0506 .0149	.0716 .0271	.0782 .0359	.0781 .0417	.0752 .0453	.0713 .0472
3	0	0074	.0142	.0201	.0250	.0290	.0320
4 5	0	.0046 .0032	.0091 .0064	.0132 .0094	.0168 .0121	.0201 .0147	.0228 .0170
5 6	0	.0024	.0047	.0069	.0090	.0110	.0128
7 8	0	.0018	.0035	.0051	.0067 .0048	.0082	.0096 .0070
9	0	.0012 .0008	.0025 .0016	.0036 .0024	.0040	.0059 .0039	.0046
10	ŏ	.0004	.0008	.0013	.0017	.0021	.0026
11.	0	.0001	.0002	.0003	•0004	.0006 0008	.0007 0009
12 13	0	0002 0005	000 <sup>1</sup> 4 0010	0006 0014	0007 0017	0021	0023
14	Ö	0007	0014	0020	0026	0031	0035
15	0	0009	0018	0025	0032	0039	0045
16	0	0010 0011	0020 0022	0029 0031	0037 0040	0045 0048	0052 0056
17 18	0 0	0012	0022	0032	0041	0050	0057

		Shear	flow, q <sub>ij</sub>	L, at stat	ion -	
j	1 = 0	i = 1	1 = 2	1=3	1 = 4	1 = 5
012345678901123145617	0.0823 .0317 .0168 .0094 .0048 .0015 0026 0039 0047 0051 0052 0049 0044 0037 0028 0017	0.0494 .0284 .0162 .0094 .0050 .0018 0022 0034 0047 0047 0041 0034 0026 0016	0.0305 .0239 .0151 .0092 .0051 .0021 0018 0030 0038 0042 0043 0041 0037 0031 0024 0015	0.0193 .0194 .0136 .0087 .0050 .0022 .0002 .0034 .0038 .0039 .0038 .0039 .0039 .0029 .0029	0.0127 .0156 .0120 .0081 .0049 .0024 .0004 0030 0036 0035 0036 0036 0036 0036 0036	0.0086 .0125 .0105 .0075 .0047 .0024 .0006000900190031003200320032002900180011

TABLE 1.- LOAD DISTRIBUTION DUE TO A UNIT CONCENTRATED PERTURBATION LOAD  $\text{ON STRINGER} \quad \mathbf{j} = \mathbf{0} \quad \text{AT STATION} \quad \mathbf{1} = \mathbf{0} - \mathbf{Concluded}$ 

 $\boxed{C = 0; m = 36}$ 

(e) B = 1,000

1		St	ringer los	d, p <sub>ij</sub> , s	t station	-	
j	1 = 0	1=1	1 = 2	i = 3	ī = #	1=5.	i = 6
0	0.5000	0.4001 .0323	0.3248 .0530	0.2676 .0657	0.2237 .0731	0.1900 .0769	0.1637 .0783
2	ŏ	.0084	.0162	.0232	.0290	.0338	.0378
3	0	.0041	.0081	.0119	.0154	.0187	.0216
4	0	.0026	.0051	•0075	.0099	.0121	.0143
5 6	0	.0018	•0036	<b>.</b> 0053	.0070	.0086	.0102
	0	.0013	.0026	•0039	.0051	.0064	.0075
7 8	0	.0010	.0019	.0029	.0038	.0047	.0056
8	0	•0007	•0014	.0020	.0027	.0033	.0040
9	0	.0004	•0009	•0013	.0018	.0022	.0026
10	0	.0002	•0004	•0007	•0009	•0012	•0014
끄	0	•0000	•0001	.0001	.0002	.0002	.0003
12	0	0001	0003	0004	0005	0006	0006
13	0	0003	0006	0008	0010	0013	0015
14	0	0004	0008	0012	0015	0019	0022
15	0	0005	0010	0015	0019	0023	0027
16	0	0006	0011	0017	0022	0027	0032
17	0	0006	0012	0018	0024	0029	0034
18	0	0006	0013	0018	0024	0030	0035

		Shear	flow, q <sub>ij</sub>	L, at stat	ion -	
3	<b>i</b> = 0	1 = 1	i = 2	1 = 3	i = 4	1 = 5
012345678901234567	0.0499 .0177 .0092 .0051 .0026 .0005 0015 0029 0029 0029 0029 0025 0010 0003	0.0377 .0170 .0092 .0052 .0027 .0009 0004 0025 0025 0026 0026 0026 0020 0015 0009 0003	0.0286 .0159 .0090 .0052 .0027 .0010000300120025002600260025002900190003	0.0219 .0145 .0087 .0051 .0028 .0011 0002 0018 0022 0024 0025 0024 0022 0018 0028	0.0169 .0131 .0083 .0050 .0028 .0011 .0001 .0016 .0021 .0023 .0024 .0023 .0020 .0017 .0013 .0008	0.0132 .0118 .0079 .0049 .0028 .0012 .0000 0009 0015 0019 0022 0022 0022 0020 0016 0012

TABLE 2.- LOAD DISTRIBUTION DUE TO A UNIT DISTRIBUTED PERTURBATION LOAD  $\text{ON STRINGER} \quad \textbf{j} \ = \ \textbf{0} \quad \text{Between Rings} \quad \textbf{i} \ = \ \textbf{0} \quad \text{AND} \quad \textbf{i} \ = \ \textbf{1}$ 

$$[C = 0; m = 36]$$

(a) 
$$B = 8$$

	. Stringer load, p <sub>ij</sub> , at station -						
j	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6	
0 1 2 3 4 5 6 7 8 9 0 1 1 2 3 4 5 6 7 8 1 1 2 3 4 5 6 7 8 1 1 2 3 4 5 6 7 8 1 1 2 3 4 5 6 7 8 1 2 3 4 5 6 7 8 8 1 2 3 4 5 6 7 8 8 1 2 3 4 5 6 7 8 8 1 2 3 4 5 6 7 8 8 1 2 3 4 5 6 7 8 8 1 2 3 4 5 6 7 8 8 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 7 8 7 8 9 1 1 2 3 4 5 7 8 7 8 7 8 9 1 1 2 3 4 7 8 7 8 7 8 7 8 7 8 7 8 7 8	0.1924 .0727 .0340 .0195 .0128 .0092 .0068 .0051 .0036 .0024 .0013 .0003 .0006 .0013 .0006 .0013 .0006 .0013 .0006	0.0567 .0629 .0499 .0391 .0226 .0175 .0134 .0067 .0038 .0013 -0010 -0047 -0060 -0076 -0076	0.0499 .0475 .0447 .0398 .0341 .0286 .0234 .0185 .0140 .0098 .0059 .0023000900370061008000940105	0.0447 .0441 .0421 .0390 .0351 .0307 .0260 .0211 .0164 .0117 .0073 .0032 0066 0040 0068 0091 0117 0121	0.0430 .0425 .0410 .0385 .0352 .0313 .0270 .0224 .0176 .0128 .0081 .0037 0004 0040 0071 0096 0114 0126	0.0423 .0418 .0404 .0382 .0352 .0316 .0274 .0229 .0134 .0086 .0040 0002 0040 0073 0138 0130	

	Shear flow, q <sub>1,j</sub> L, at station -							
j	i = 0	1 = 1	i = 2	1 = 3	1 = 4	i = 5		
01234567890112345617	0.3077 .1623 .0942 .0553 .0296 .0112 0025 0127 0200 0248 0273 0279 0268 0240 0201 0151 0094 0032	0.0679 .0776 .0617 .0433 .0271 .0137 .0030 0053 0115 0183 0183 0189 0172 0145 0109 0068 0023	0.0034 .0188 .0240 .0221 .0171 .0110 .0052 .0001 0041 0072 0093 0104 0105 0098 0084 0064 0040	0.0026 .0060 .0086 .0094 .0085 .0064 .0038 .0012 0012 0045 0053 0056 0054 0047 0036 0023 0008	0.0008 .0025 .0036 .0041 .0039 .0033 .0022 .0010 0002 0013 0021 0027 0029 0028 0025 0020 0012	0.0004 .0016 .0018 .0018 .0016 .0012 .0006 .0000 0005 0010 0013 0015 0015 0010 0007 0007		

TABLE 2.- LOAD DISTRIBUTION DUE TO A UNIT DISTRIBUTED PERTURBATION LOAD ON STRINGER j=0 Between Rings i=0 AND i=1- Continued

$$C = 0; m = 36$$

(b) B = 30

	Stringer load, p <sub>ij</sub> , at station -							
j	i = 1	i = 2	i = 3	1 = 4	i = 5	i = 6		
0 1 2 3 4 5 6 7 8 9 0 1 1 2 3 4 1 5 6 1 7 1 8	0.2850 .0603 .0214 .0111 .0071 .0050 .0037 .0027 .0013 .0007 .0001 0004 0008 0011 0014 0016 0017	0.1124 .0789 .0447 .0278 .0191 .0139 .0104 .0056 .0057 .0020 .0005 0020 0020 0029 0043 0046	0.0729 .0644 .0489 .0360 .0270 .0206 .0159 .0120 .0088 .0059 .0034 .0010 0028 0043 0043 0069 0069	0.0583 .0550 .0473 .0388 .0313 .0251 .0198 .0154 .0014 .0046 .0016 0010 0033 0052 0068 0079 0086	0.0514 .0496 .0452 .0394 .0278 .0226 .0178 .0134 .0094 .0056 .0022 0009 0036 0059 0077 0091	0.0476 .0465 .0435 .0393 .0345 .0294 .0196 .0150 .0106 .0065 .0027 0008 0038 0064 0084 0099 0108		

3	Shear flow, q <sub>ij</sub> L, at station -							
J	1 = 0	i = 1.	i = 2	i = 3	1 = 4	i = 5		
012345678901123456	0.2150 .0944 .0516 .0294 .0152 .0052 0022 0076 0115 0140 0154 0156 0149 0124 0084 0084	0.0863 .0678 .0444 .0277 .0156 .0067 .0000 0051 0087 0112 0125 0129 0124 0112 0094 0094	0.0198 .0343 .0301 .0220 .0141 .0074 .0020 0023 0055 0077 0090 0095 0094 0085 0072 0054	0.0073 .0167 .0183 .0155 .0111 .0067 .0027 0006 0033 0052 0064 0070 0070 0064 0055 0042	0.0035 .0088 .0109 .0103 .0082 .0054 .0027 .0002 0019 0034 0045 0050 0051 0048 0041 0031	0.0019 .0050 .0066 .0067 .0057 .0041 .0023 .0005 0010 0022 0031 0036 0037 0035 0030 0023		
17	0018	0015	0012	0009	0007	- 0005		

TABLE 2.- LOAD DISTRIBUTION DUE TO A UNIT DISTRIBUTED PERTURBATION LOAD ON STRINGER j = 0 Between RINGS i = 0 AND i = 1 - Continued  $\boxed{C = 0; m = 36}$ 

(c) B = 100

	Stringer load, p <sub>ij</sub> , at station -							
j	1 = 1	1 = 2	i = 3	1 = 4	1 = 5	i = 6		
0 1 2 3 4 5 6 7 8 9 0 1 1 2 3 4 5 6 1 7 8 1 1 2 3 4 5 6 1 7 8	0.3600 .0428 .0127 .0063 .0040 .0028 .0020 .0015 .0011 .0007 .0004 .0001 0002 0004 0006 0008	0.1972 .0770 .0324 .0177 .0115 .0081 .0060 .0044 .0032 .0021 .0011 .0002 0005 0018 0028	0.1268 .0779 .0431 .0264 .0179 .0129 .0096 .0072 .0051 .0034 .0019 .0005 0007 0018 0027 0035 0040 0043 0043	0.0932 .0714 .0474 .0321 .0229 .0170 .0129 .0097 .0070 .0047 .0026 .0008 0009 0023 0035 0045 0052	0.0753 .0645 .0485 .0357 .0267 .0204 .0157 .0119 .0087 .0059 .0033 .0011 0010 0027 0042 0054 0068 0068	0.0648 .0589 .0480 .0377 .0295 .0231 .0181 .0138 .0102 .0070 .0040 .0014 0010 0030 0048 0062 0072 0078		

_	Shear flow, q <sub>ij</sub> L, at station -							
j t	1 = 0	1 = 1	1 = 2	i = 3	i = 4	1 = 5		
0 1 2 5 4 5 6 7 8 9 10 11 2 3 4 5 6 17	0.1400 .0543 .0289 .0162 .0082 .0026 0015 0045 0080 0087 0089 0087 0089 0064 0048 0048	0.0814 .0472 .0275 .0161 .0086 .0032 0007 0056 0071 0078 0080 0077 0069 0058 0014 0027	0.0352 .0343 .0237 .0151 .0087 .0039 .0002 0045 0058 0066 0060 0060 0050 0038 0024 0008	0.0168 .0234 .0191 .0133 .0082 .0041 .0008 0035 0048 0056 0056 0051 0043 0033 0020 0007	0.0090 .0158 .0148 .0112 .0074 .0040 .0012 0010 0027 0039 0046 0049 0048 0044 0037 0028 0018	0.0052 .0108 .0113 .0092 .0065 .0038 .0014 0020 0031 0038 0041 0041 0038 0032 0024 0015 0005		

TABLE 2.- LOAD DISTRIBUTION DUE TO A UNIT DISTRIBUTED PERTURBATION LOAD ON STRINGER j=0 BETWEEN RINGS i=0 AND i=1 - Continued  $\boxed{C=0; \ m=36}$ 

(d) B = 300

<del></del>	<del></del>					
j		Strin	ger load,	p <sub>ij</sub> , at sta	tion -	
	1 = 1	i = 2	i = 3	1 = 4	1 = 5	<b>i</b> = 6
012345678901123456178	0.4108 .0286 .0076 .0037 .0023 .0016 .0012 .0009 .0006 .0004 .0002 .0000 0001 0003 0004 0005 0006	0.2820 .0628 .0213 .0109 .0069 .0048 .0036 .0026 .0018 .0012 .0006 .0001 0004 0007 0011 0014 0015 0017	0.2038 .0757 .0318 .0172 .0111 .0079 .0058 .0043 .0031 .0020 .0010 .0002 0005 0012 0017 0022 0027	0.1549 .0785 .0390 .0227 .0150 .0108 .0080 .0059 .0042 .0028 .0015 .0003 0007 0016 0023 0029 0036 0037	0.1234 .0768 .0436 .0271 .0185 .0134 .0100 .0074 .0054 .0035 .0019 .0005 0008 0019 0028 0036 0041 0046	0.1026 .0733 .0464 .0305 .0215 .0158 .0119 .0089 .0064 .0043 .0024 .0006 0009 0022 0033 0042 0048 0052

j		Shear flow, q <sub>ij</sub> L, at station -							
	i = 0	i = 1	i = 2	1 = 3	1 = 4	1 = 5			
0 1 2 3 4 5 6 7 8 9 9 1 1 2 3 4 5 6 17	0.0892 .0320 .0168 .0094 .0014 0010 0027 0040 0048 0052 0052 0050 0045 0037 0028 0017 0006	0.0644 .0302 .0166 .0094 .0017 0007 0024 0036 0049 0049 0049 0042 0035 0026 0016	0.0391 .0262 .0157 .0093 .0050 .0020 0032 0040 0045 0045 0043 0039 0033 0024 0015	0.0244 .0216 .0144 .0089 .0050 .0022 .0000 0016 0028 0036 0040 0041 0040 0036 0030 0023 0014 0005	0.0157 .0175 .0128 .0084 .0050 .0023 .0003 0013 0036 0037 0036 0037 0036 0037 0036 0037	0.0104 .0140 .0113 .0078 .0048 .0024 .0005 0010 0021 0028 0032 0034 0033 0030 0025 0019 0012			

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TABLE 2.- LOAD DISTRIBUTION DUE TO A UNIT DISTRIBUTED PERTURBATION LOAD ON STRINGER j=0 Between RINGS i=0 AND i=1 - Concluded  $\boxed{C=0;\ m=36}$ 

(e) B = 1,000

	Stringer load, p <sub>ij</sub> , at station -							
j	i = 1	i = 2	1 = 3	1 = 4	1=5	i = 6		
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 6 17	0.4477 .0173 .0042 .0021 .0013 .0009 .0007 .0005 .0003 .0002 .0001 .0000 0001 0002 0003	0.3607 .0434 .0124 .0061 .0038 .0027 .0020 .0014 .0010 .0007 .0003 .0000 0004 0004 0008	0.2949 .0599 .0198 .0100 .0063 .0044 .0032 .0024 .0017 .0011 .0006 .0001 0003 0007 0010	0.2447 .0698 .0262 .0137 .0087 .0061 .0045 .0033 .0024 .0015 .0008 .0002 0004 0009 0014	0.2061 .0752 .0315 .0171 .0110 .0078 .0058 .0042 .0030 .0020 .0010 .0002 0005 0012 0017 0021	0.1763 .0777 .0359 .0202 .0132 .0094 .0070 .0051 .0037 .0024 .0013 .0003 0006 0014 0020 0029		
17 18	0003 0003	0009 0010	0015 0016	0021 0021	0026 0027	0032 0032		

	Shear flow, q <sub>ij</sub> L, at station -							
3	i = 0	i = 1	1 = 2	i=3	1 = 4	i = 5		
0 1 2 3 4 5 6 7 8 9 0 1 1 2 3 4 5 6 17	0.0523 .0177 .0092 .0051 .0026 .0006 0015 0022 0027 0029 0029 0029 0025 0021 0016 0010	0.0435 .0174 .0092 .0052 .0026 .0005 0014 0021 0026 0028 0028 0027 0024 0020 0015 0009	0.0329 .0164 .0091 .0052 .0027 .0009 0013 0020 0024 0026 0027 0026 0023 0019 0014 0009	0.0251 .0152 .0088 .0051 .0027 .0010 0002 0018 0025 0026 0024 0024 0018 0014 0008	0.0193 .0138 .0085 .0051 .0028 .0011 0010 0017 0021 0024 0023 0021 0013 0013 0008	0.0149 .0124 .0081 .0050 .0028 .0012 .0000 0016 0020 0022 0023 0022 0021 0017 0013 0008		

TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERTURBATION LOAD

ABOUT SHEAR PANEL (0,0)

$$[c = 0; m = 36]$$

(a) B = 8

j	Stringer load, p <sub>ij</sub> /L, at station -								
	i = 1.	i = 2	i = 3	1 = 4	1 = 5	i = 6			
1 2 3 4 5 6 7 8 9 10 11	-0.1192 0374 0125 0038 0002 .0016 .0026 .0032 .0036 .0037	0.0067 0118 0100 0061 0031 0011 .0003 .0011 .0017	-0.001900160029002100120005 .0002 .0006 .0009	-0.0001 0008 0010 0011 0010 0007 0004 0001 .0002	-0.0001 0003 0004 0005 0005 0004 0002 0001 .0000	0.0000 0001 0002 0002 0002 0001 0001 .0000			
12 13 14 15 16 17 18	.0035 .0032 .0032 .0028 .0023 .0017 .0010	.0021 .0020 .0018 .0014 .0011 .0007	.0012 .0011 .0010 .0009 .0007 .0004	.0006 .0006 .0006 .0005 .0004 .0002	.0002 .0003 .0003 .0003 .0003 .0002 .0001	.0001 .0002 .0002 .0002 .0001 .0001			

1 3	Shear flow, q <sub>ij</sub> , at station -							
,	<b>i =</b> 0	1=1	i = 2	1 = 3	i = 4	1 = 5		
14 15 16	0.6986 0629 .0119 .0370 .0446 .0451 .0419 .0366 .0302 .0230 .0156 .0082 .0012 0052 0108 0153 0167	0.1357 .0097 0159 0184 0162 0134 0107 0083 0062 0043 0025 0010 .0004 .0017 .0027 .0035 .0045	0.0068 .0154 .0052001900510060005800510042002100110001 .0007 .0014 .0020	0.0052 .0034 .0026 .0008 0010 0021 0026 0024 0019 0014 0008 0003 .0002 .0007 .0011	0.0016 .0016 .0011 .0005 0001 0010 0012 0011 0008 0006 0002 .0000 .0003	0.0008 .0007 .0005 .0000 0002 0004 0006 0006 0004 0003 0002 .0000 .0002		

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TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERTURBATION LOAD

ABOUT SHEAR PANEL (0,0) - Continued

$$\boxed{C = 0; m = 36}$$

(b) B = 30

	Stringer load, p <sub>ij</sub> /L, at station -						
J	1 = 1	i = 2	i = 3	i = 4	i = 5	i = 6	
1 2	-0.2242 0377	-0.0331 0329	-0.0081 0142	-0.0029 0064	-0.0013 0032	-0.0007 0017	
3 4	0082 0012	0148 0060	0109 0062	0064 0047	0037 0032	0022 0021	
5	.0013	0017	0029 0008	0028 0012	0022 0012	0016 0010	
7 8	.0034	.0017	.0006 .0014	0001 .0007	0004 .0003	0004	
9	.0042 .0042	.0029 .0031	.0020 .0023	.0012	.0008	.0004 .0007	
11	.0041 .0039	.0032	.002 <sup>1</sup> 4 .0023	.0017	.0012 .0013	.0009	
13 14	.0036	.0028	.0022 .0019	.0017 .0015	.0013 .0011	.0009	
15 16	.0025	.0020 .0015	:0016 :0012	.0012	.0010 .0007	.0007 .0006	
17 18	.0011 .0004	.0009 .0003	.0007 .0002	.0006 .0002	•000 <sup>4</sup> •0002	.0003 .0001	

		Shear	r flow, q <sub>i</sub>	j, at stati	on -	
j	1 = 0	i = 1	1 = 2	i = 3	1 = 4	i = 5
01234567890112345617	0.5133 0382 .0372 .0537 .0562 .0535 .0482 .0413 .0336 .0253 .0168 .0085 .0085 .0007 0126 0176 0212	0.1726 0186 0233 0167 0120 0089 0067 0050 0037 0024 0004 0005 0012 0018 0023 0029	0.0395 .0145 .0042 .0081 .0079 .0054 .0042 .0032 .0022 .0013 .0005 .0008 .0013 .0018	0.0146 .0094 .0016 0028 0043 0040 0033 0026 0019 0012 0006 .0005 .0005 .0010	0.0070 .0054 .0021 0006 0022 0027 0028 0025 0020 0016 0010 0006 0001	0.0038 .0031 .0016 .0001 0010 0018 0018 0015 0012 0009 0005 0001

TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERTURBATION LOAD

ABOUT SHEAR PANEL (0,0) - Continued

$$[C = 0; m = 36]$$
(c) B = 100

	Stringer load, p <sub>ij</sub> /L, at station -						
Ĵ	i = 1	1 = 2	i = 3	1 = 4	i = 5	1 = 6	
1 2 3 4 5 6 7 8 9 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8	-0.316802880043 .0004 .0022 .0032 .0045 .0045 .0045 .0041 .0037 .0032 .0026 .0019 .0012	-0.1198043301270035 .0001 .0018 .0028 .0034 .0037 .0039 .0038 .0036 .0033 .0028 .0023	-0.04850335014700570015 .0007 .0019 .0027 .0033 .0033 .0032 .0029 .0025 .0020 .0015	-0.021402260132006400250002 .0012 .0025 .0028 .0028 .0027 .0025 .0028 .0027 .0025	-0.0103 0148 0107 0062 0029 0008 .0006 .0015 .0023 .0024 .0024 .0024 .0022 .0019 .0016 .0012	-0.005500970085005400290011 .0002 .0016 .0019 .0020 .0019 .0020 .0017 .0014 .0006	

j		Shear	r flow, q <sub>1</sub>	j, at stati	on -	
,	<b>i</b> = 0	i = 1	i = 2	i = 3	i = 4	1=5
01234567890112345678	0.3632 0032 .0546 .0632 .0624 .0579 .0515 .0438 .0353 .0264 .0174 .0086 .0004 0071 0135 0188 0226 0250 0258	0.162803420197011400750053004000290021001400080002 .0003 .0008 .0011 .0014 .0017 .0018	0.0704 0009 0106 0086 0054 0036 0027 0020 0013 0008 0002 0002 0006 0010 0010 0016 0016	0.0337 .0065 0043 0058 0051 0032 0032 0018 0013 0008 0003 .0001 .0005 .0001 .0008 .0011	0.0179 .0068 0010 0035 0038 0028 0022 0017 0012 0007 0003 .0001 .0004 .0007 .0009 .0011	0.0104 .0056 .0005 0020 0027 0027 0024 0015 0011 0007 0003 .0000 .0006 .0008 .0008

TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERTURBATION LOAD

ABOUT SHEAR PANEL (0,0) - Continued

$$[0 = 0; m = 36]$$

(d) 
$$B = 300$$

	Stringer load, p <sub>ij</sub> /L, at station -						
j	i = 1	1 = 2	1 = 3	1 = 4	i = 5	<b>1</b> = 6	
1 2 7 4 5 6 7 8 9 9 1 1 2 3 4 5 6 7 8	-0.3817 -0197 -0018 -0014 -0027 -0035 -0041 -0046 -0046 -0046 -0048 -0038 -0033 -0027 -0020 -0004	-0.2188 -0.402 -0.084 -0.014 -0.027 -0.034 -0.039 -0.042 -0.042 -0.039 -0.042 -0.039 -0.042	-0.1276042701250033 .0002 .0019 .0034 .0038 .0039 .0038 .0036 .0037 .0029 .0023	-0.0759038301430049008 .0012 .0023 .0030 .0034 .0035 .0035 .0031 .0027 .0022 .0016 .0010	-0.04620319014500580016 .0006 .0018 .0026 .0030 .0032 .0032 .0031 .0029 .0025 .0020 .0015	-0.02880256013800630022 .0000 .0014 .0029 .0029 .0026 .0023 .0019 .0019 .0009	

j	Shear flow, q <sub>ij</sub> , at station -						
ال	i = 0	i = 1	1 = 2	i = 3	1 = 4	1=5	
0 1 2 3 4 5 6 7 8 9 0 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 1 2 3 4 5 6 7 8 9 1 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 1 2 3 4 5 6 7 8 9 1 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 1 2 3 4 5 6 7 8 9 1 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9 1 1 2 3 4 5 6 7 8 9	0.2618 .0253 .0648 .0657 .0602 .0532 .0450 .0362 .0270 .0177 .0087 .0002 0140 0194 0234 0258	0.1287 0341 0137 0071 0045 0024 0017 0012 0008 0004 0001 .0002 .0005 .0007 .0009 .0010	0.0782 0130 0105 0064 0043 0031 0021 0017 0012 0008 0004 0001 .0002 .0004 .0006 .0008 .0009	0.0489 0028 0072 0054 0039 0029 0016 0012 0008 0004 0001 .0002 .0004 .0006 .0007 .0009 .0009	0.0315 .0017 0046 0044 0027 0020 0015 0011 0008 0004 0001 .0003 .0005 .0007 .0008 .0008	0.0209 .0035 0027 0035 0030 0019 0015 0011 0007 0004 0002 .0001 .0003 .0005 .0006 .0007	

TABLE 3.- LOAD DISTRIBUTION DUE TO A UNIT SHEAR PERFURBATION LOAD

ABOUT SHEAR PANEL (0,0) - Concluded

$$[C = 0; m = 36]$$

(e) B = 1,000

		String	er load, p	ij/L, at st	ation -	
j	i = 1	i = 2	i = 3	i = 4	i = 5	i = 6
1 2 3 4 5 6 7 8 9 1 1 1 2 1 3 1 4 1 5 1 6 1 7 1 8	-0.450001180002 .0020 .0030 .0037 .0042 .0047 .0047 .0046 .0043 .0039 .0034 .0027 .0020 .0012	-0.316902970043 .0005 .0023 .0032 .0039 .0042 .0045 .0045 .0041 .0038 .0032 .0026 .0019 .0012	-0.2346038800770009 .0015 .0028 .0035 .0040 .0042 .0043 .0040 .0036 .0031 .0025 .0019 .0011	-0.1745042401040022 .0008 .0023 .0037 .0040 .0041 .0040 .0038 .0035 .0030 .0024 .0011 .0004	-0.1305 0425 0124 0033 .0002 .0019 .0029 .0035 .0038 .0039 .0036 .0033 .0029 .0024 .0017 .0011	-0.0981 0406 0136 0042 0004 .0015 .0026 .0032 .0037 .0037 .0037 .0037 .0039 .0023 .0017 .0010 .0004

3		Shear	flow, q <sub>i</sub>	j, at stati	on -	
J	1 = 0	i = 1	1 = 2	i = 3	1 = 4	1 = 5
01234567890112345678	0.1879 .0479 .0715 .0718 .0678 .0617 .0542 .0458 .0367 .0273 .0179 .0088 .0001 0076 0144 0198 0238 0263 0271	0.0870 0261 0082 0040 0025 0018 0013 0010 0007 0004 0002 .0000 .0001 .0003 .0004 .0005 .0006 .0006	0.065801650074003900150018001000070004000200010003000400060006	0.0502 0099 0064 0037 0013 0009 0007 0004 0002 .0000 .0001 .0002 .0004 .0005 .0005	0.038600550054003400230017001200090007000400020001 .0001 .0002 .0003 .0004 .0005 .0005	0.029900250014003100220016001200090006000400020001 .0001 .0002 .0003 .0003 .0005

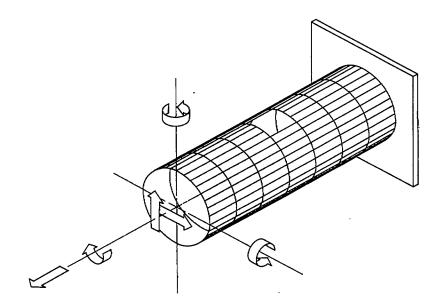
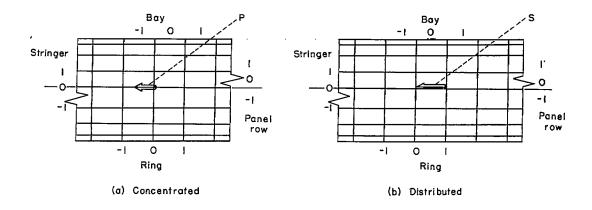


Figure 1.- Circular semimonocoque cylinder with cutout.



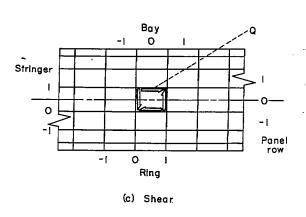


Figure 2.- Perturbation loads.

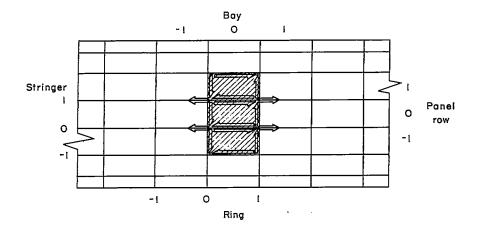


Figure 3.- Application of perturbation loads.

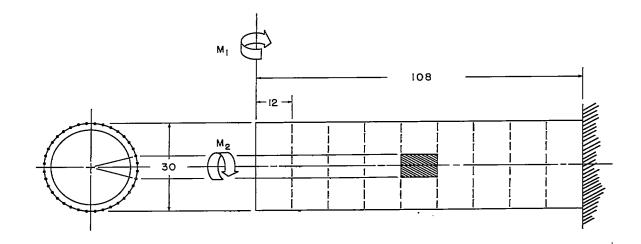
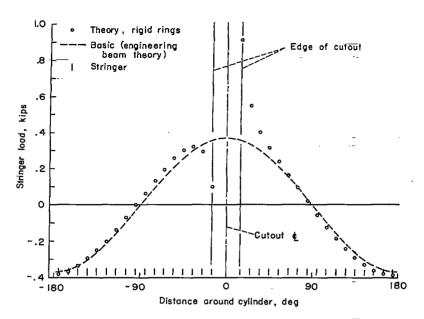
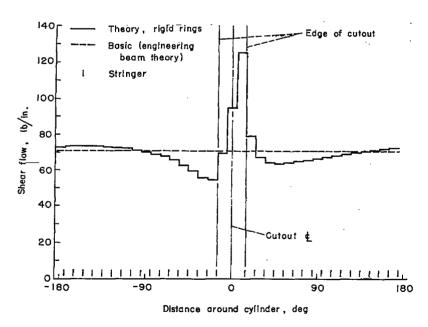


Figure 4.- Circular cylinder with cutout used in sample calculation.



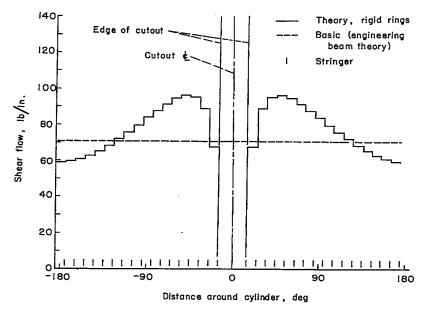
(a) Stringer loads at ring bordering cutout (ring  $i=\overline{O}$ ).

Figure 5.- Results of sample calculation.



(b) Shear flow in bay adjacent to cutout (bay I=-1).

Figure 5.- Continued.



(c) Shear flow in net section (bay i=0).

Figure 5.- Concluded.

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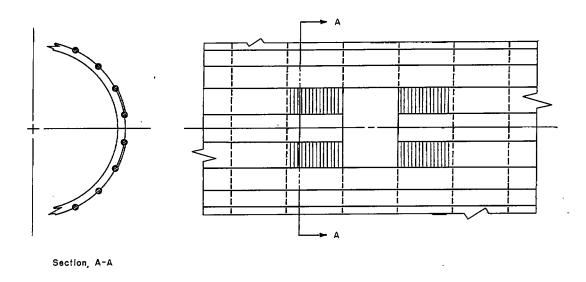


Figure 6.- Cutout with shear reinforcement.

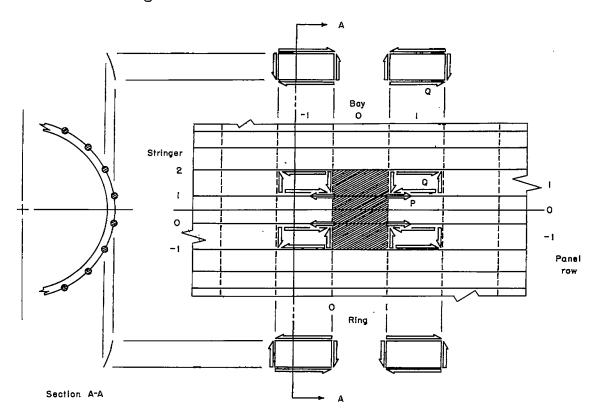


Figure 7.- Perturbation load system for a problem of shear reinforcement.

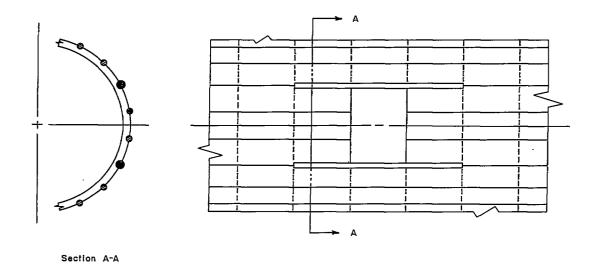


Figure 8.- Cutout with reinforced coaming stringers.

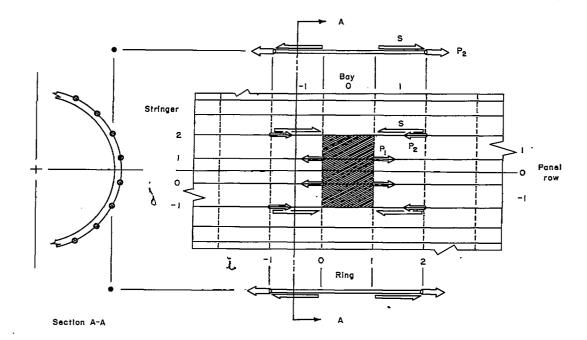


Figure 9.- Perturbation load system for a problem of coaming-stringer reinforcement.